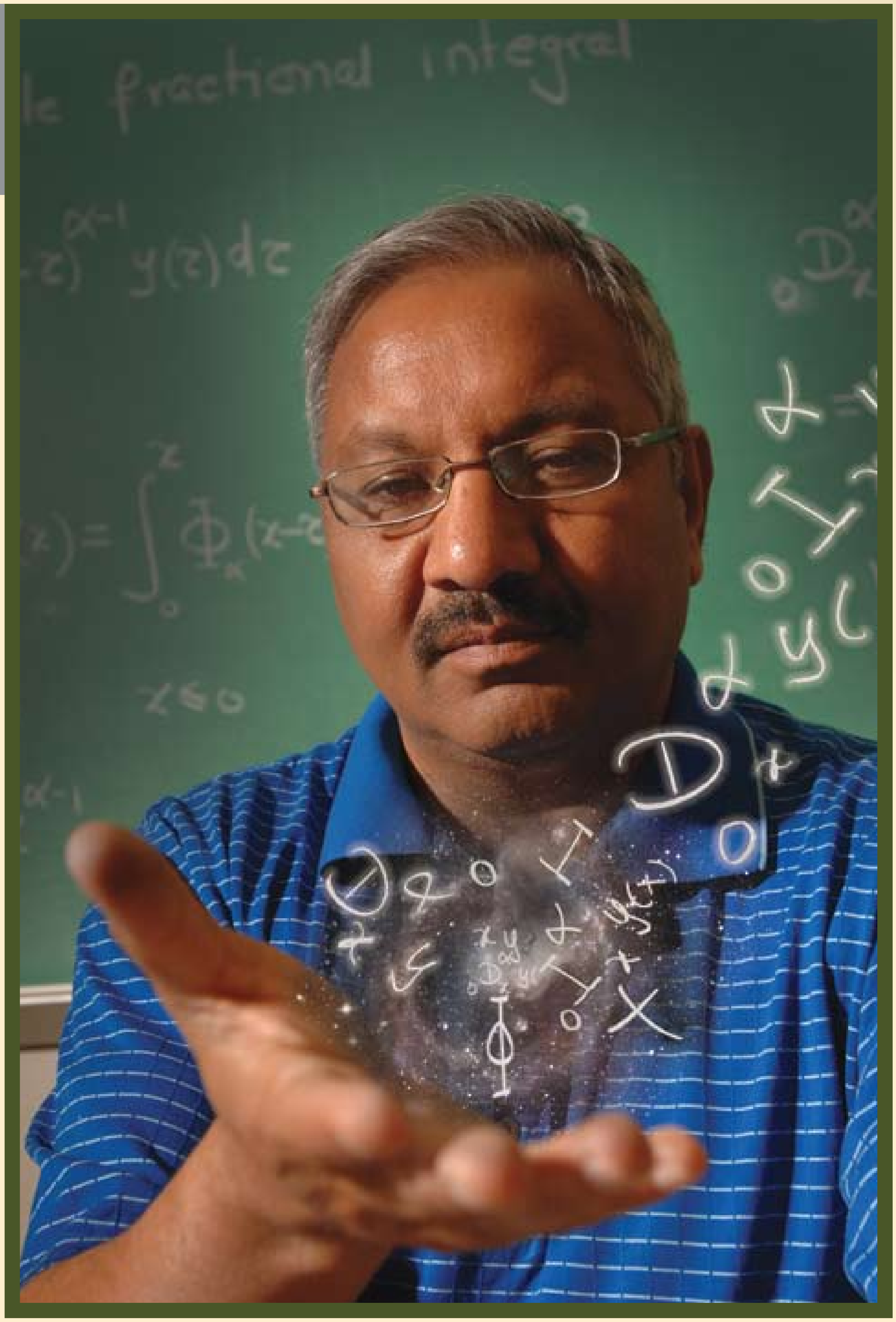
CUTTING-EDGE MATH

A seldom-taught branch of calculus provides better tools for real-world challenges, says Om Agrawal

by Marilyn Davis



Mechanical engineer Om Agrawal doesn't consider himself a mathematician. He doesn't number himself among those who expand frontiers in the high realms of pure mathematics. But, working in the area of applied mathematics, he has gained attention as a leading exponent of an offshoot of calculus increasingly useful in engineering and science.

"Fractional calculus is the calculus of the future," says Agrawal, who has co-organized several international symposia on this branch of math and its applications. "With it, we can solve problems we couldn't have solved before."

Fractional calculus often better describes the behavior of complex systems and materials than traditional calculus does, he says. With tools from fractional calculus, researchers are developing new mathematical models in fields from bioengineering to economics.

French researchers have designed improved shock absorbers using fractional calculus. A colleague of Agrawal's at the University of Illinois at Chicago has demonstrated that fractional calculus gives more accurate interpretations of nuclear magnetic resonance data in imaging the brain. Colleagues at the University of Akron and NASA have found that fractional calculus models better describe the movement and structure of hurricanes and galaxies.

Agrawal himself was the first researcher to use fractional calculus to develop theories and numerical schemes for optimal controllers for mechanical systems.

The idea of fractional calculus was raised by the mathematicians who first developed traditional calculus. But not until about 30 or so years ago, when certain mathematical issues had been worked out, did engineers and scientists begin using it to tackle real-world problems. It has since found many applications—yet it is greatly underused and underappreciated, Agrawal says.

Fractional calculus involves derivatives and integrals of so-called fractional order. Derivatives are what tell you the rate of change in a system. Take the case of a moving car. Velocity tells you how distance is changing with time; acceleration tells you how velocity is changing with time. Velocity is the first-order derivative of distance with respect to time; acceleration is the second-order derivative of distance with respect to time.

"Currently most of the engineering problems we solve are formulated in terms of integer-order derivatives: first, second, and so forth," Agrawal says. "And these are local derivatives, meaning that you only need local values.

"For example, if you want to know your [car's] acceleration at approximately 10 seconds after you start driving, you only need to know your velocity at around 10 seconds, not what it was at 2 seconds or 5 seconds." Solving such problems doesn't depend on the *history* of the system.

But many natural and manmade systems—from body tissues to climate effects to viscoelastic materials such as polymers and gels—"react not only based on their current state, but also on the previous states they have gone through," Agrawal notes. Because integer-order derivatives are local values, using them to predict such systems' behavior requires elaborate models and often employs an averaging approach to try to handle irregularities and discontinuities.

Fractional derivatives provide a better way, Agrawal says. The simplest lay explanation is that fractional derivatives capture intermediate states; mathematically, they incorporate the history of the system or material. Although models based on fractional derivatives require much more data—information about previous states—the model itself can be much simpler.

Agrawal tries to "hypothesize and predict" fractional calculus approaches that could be used by colleagues to solve engineering problems. They in turn have the hard data about systems that he needs to verify his mathematical formulas.

One area where Agrawal has used fractional derivatives is in thermal analysis of disc brakes, crucial to developing longer-lasting brake materials. In the past, predicting the surface temperature distribution of a disc involved a very complicated model. However, Agrawal says, "Under certain assumptions the surface temperature can be predicted very easily using fractional derivatives." (Engineers working on a system "have a good idea what assumptions will hold up," he says, though testing is needed to confirm them.)

Fractional derivatives are a "cutting-edge tool" for the design and analysis of materials, Agrawal says. Many of the materials suited to fractional calculus models have not been fully characterized, he adds: "Many of their behaviors are not fully known."

Using fractional calculus, researchers are developing improved mathematical models in fields from bioengineering to economics.

Agrawal's work on optimal controllers has applications to mechanical, electrical, and other systems, including robotics. He pioneered equations for optimal control of systems whose behavior fits fractional-order models and also is subject to random inputs.

In addition, he was one of the first researchers to incorporate fractional derivatives in what's called variational calculus, developed for problems that involve nature's tendency to minimize potential energy in systems. "My idea was that if a system requires fractional derivatives to define its behavior, there must be an energy-minimizing law in the system coming into the picture," he says.

"Fractional variational calculus" remains highly theoretical, he says: "The future will tell where the applications are." But other researchers have been using it to address abstract problems in mechanics and have dubbed one of his formulations "Agrawal's Principle."

Agrawal has many U.S. and international collaborators, in disciplines as diverse as physics and engineering. In 2004 he led a seven-member interdisciplinary U.S. team to Bordeaux on a National Science Foundation–sponsored effort to work with a similar French team. The goal: to see how their expertise in fractional calculus could be combined to solve problems in various areas.

Also in 2004, French colleague Jocelyn Sabatier, Portuguese colleague J. A. Tenreiro Machado, and Agrawal co-edited a special issue of the journal *Nonlinear Physics.* In 2007, they worked together again on a book called *Advances in Fractional Calculus: Theoretical Developments and Applications in Physics and Engineering* (Springer).

In 2006 Agrawal was recognized for his career contributions at an international conference on mathematical methods in engineering at Ankara, Turkey, where he gave a keynote speech and other presentations. He will speak at the 2008 meeting this fall, as well as at other venues—as always, advocating for fractional calculus as a valuable tool to understand our complex world.

Dr. Om Agrawal, Dept. of Mechanical Engineering and Energy Processes, was named the College of Engineering's Outstanding Scholar in 2007. For more information about his research, contact him at om@engr.siu.edu or see http://meep.engr.siu.edu/faculty/agrawal/index.htm.